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MATHEMATICAL MODELLING AND EXPERIMENTATION OF SOY WAX 68℃ ENCAPSULATED INTO SOLAR TANK (CASE STUDY)

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Abstract: Governments worldwide are attempting to minimise CO_2 emissions, and solar energy storage remains the most difficult problem to tackle in the current climate. Typically, domestic hot water is mainly used for heat process services in colder climates when the tank loss is significant. Any design modification can result in a higher solar yield. Since water is a perfect medium for heat storage, this article will examine different Solar Domestic Hot Water (SDHW) systems in cold climates such as Central Europe and hot and dry climates such as the Middle East (Syria). Linear modelling was conducted using R script software using coded values to define the optimal value using the response surface method (RSM). The programming phase uses the least-squares approach to provide a general rationale for the line's best-match position among the data points under consideration. For each variable, the coded values range from [-1, +1]. The number of experiments is determined by the formula 2^k, where k is the variable number. Since each variable has two possible values [-1, +1], the total number of experiments was $2^3= 8$. In addition to these experiments, we performed one more experiment for defining second-degree non-linear coefficients. The second-degree factors were checked to evaluate the non-linearity of the system. The experimental work was done in the laboratory; an insulated water tank filled with 5 litres of water was used. A capsulated PCM soy wax 68℃ test was conducted. For the charging phase, the response surface approach with non-linear correlation was used to determine the optimal number of samples and PCM quantity at two temperature levels. The results of temperature, sample numbers, and wax quantity demonstrate a 0.22, 2.3, and -1.12 first-degree magnitude effect, respectively. In addition, each two-factor interaction contour plot is depicted.

Keywords: solar tank; PCM material; thermal storage; linear modelling; R-script

1. Introduction

The rapid expansion and technological revolution of human civilisation in the last few decades has brought with it unavoidable consequences like over-exploitation of the environment and natural resources. All of this compelled the researchers to seek out a more long-term answer while still optimising the existing ones. While fossil fuels are currently the dominant energy source, resulting in large carbon dioxide emissions, environmentally friendly alternatives, such as renewable energy systems, have begun to emerge with extended life spans, high reliability, and efficiency [1, 2]. Those systems became a viable alternative to the previous ones. Renewable energy is being used not only because of CO_2 emissions from traditional fuels and because it is environmentally benign but also because of rising energy demand and fluctuating oil prices.

Renewable technologies can supply consistent, self-sufficient electricity to remote places. For example, the willingness to support new renewable energy projects grew in Hungary since it proved to be a good financial investment with a payback period of less than ten years [3, 4]. It became evident that using renewable energy sources is the best approach to comply with the Kyoto Protocol. According to data given by ESTIF (European Solar Thermal Industry Organisation) [5], solar heating and cooling have proven to be a driver even in adverse economic times, with a solar power output of 26.3 [GWth] and a surface area of more than 37 million square meters.

Water heating accounts for about 15% of total energy use in cold locations (such as Budapest). Furthermore, many applications demand temperatures of less than 100 degrees Celsius, which can be easily achieved using renewable energy, such as solar domestic water heating (SDWH) technology [6, 7]. Traditional water heating systems that use fossil fuels or electric heaters have a greenhouse effect, require a lot of maintenance, and are costly. As a result, the future of solar water heating system technology seems bright because of its environmentally benign nature and use of renewable energy [8, 9].

SDWH can be broken down into two sections: design and operation. The collectors can be made in a variety of ways. Solar collectors are divided into flat plate collectors (FPC) and evacuated tube collectors (ETC). In evacuated tube collectors, temperatures can reach 250 degrees Celsius, and efficiency can reach 80%. However, these changes depend on the weather, the design, and the load profile [10].

Some studies model the internal temperature conditions of an outdoor container based on measured meteorological data. This model can be used to predict the internal temperature of an office container, thereby reducing unnecessary environmental impact. [11, 12]

Other authors have validated a mathematical model with measured data from a real solar heating system. The error of the system model they developed is 3.4%. Similar modelling can be of great help in improving energy efficiency. [13]

The latent heat storage material is another technique to include the PCM. They are most typically employed in the solar system as a cooling solution for PV panels, which boosts their efficiency because the chemical bonds of the material break when it changes its phase. Thermal collectors can also be utilised as a storage material [14] to boost the solar system's energy capacity. One of the major advantages of integrated PCMs in solar systems is that they are simple to install and have no complicated components [3]. In addition to a high storage density [15]. As a result, any research in this field can potentially improve the tank's efficiency while reducing its size [16].

Energy is typically stored and retrieved as sensible heat, latent heat, or a thermo-chemical reaction, all of which are accomplished through a change in the medium's internal power. Sensible heat storage (SHS) stores heat by raising the temperature of the medium. When the storage medium changes phase from solid to liquid, liquid to gas, or vice versa, latent heat storage (LHS) uses the absorbed/released heat. The PCM is a heat-absorbing latent storage medium that maintains an almost constant temperature. This process will continue until the entire PCM has been transported to the liquid phase. The PCM begins to harden as the ambient temperature drops, releasing the accumulated latent heat. Melting temperatures typically vary from -50 to 1900 degrees Celsius. [17]

Some PCMs are highly beneficial within the human comfort zone (20-300°C), allowing 5 to 14 times higher heat density storage than traditional storage mediums like water or rocks [12–14]. Using a response surface method RSM, the numerical model of the entire system is used to select appropriate operational parameters and optimise the stored energy. The solar system's storage system is a crucial component and element. Furthermore, in order to maximise solar production, storage density (the amount of energy stored per volume or mass), appliance efficiency (solar collectors, tanks, and so on), and demand consumption [18], are critical aspects in determining the storage tank's capability for PCM [19], Due to the irregular nature of solar energy, this could be effective in meeting energy demand. Whereas several simulation attempts have been carried out to determine the performance of water storage tanks with PCM [20], there are no references to model the optimisation of the working variables [16], because it causes 2^k experiments to run, where k is the number of factors.

2. Materials and Methods

We performed mathematical modelling using the response surface method and experimented with the soybean wax PCM integrated into the solar tank in the laboratory. A capsulated PCM soy wax 68ºC in a 5-litre insulated water tank was conducted. The response surface approach with non-linear correlation was used for the charging phase to determine the appropriate number of samples and the quantity of PCM at two temperature levels. The method will illustrate the first-degree effect of the temperature, sample numbers, and wax quantity. Furthermore, each two-factor interaction contour plot is depicted.

The experiment components are as follows: water tank, heater, sensors, datalogger (ALMEMO 2890-9), and Wax 68°C type. The system comprises a well-insulated water tank covered by 5 cm of expanded polystyrene (EPS), an oil-based insulation material that acts as a perfect insulator in the form of foam. This foam is a non-degradable, environmentally friendly material that maintains its qualities throughout time.

The tank has dimensions of 42x13x16 cm and can hold up to 8.7 litres of water, as shown in Figure 1; however, it was only filled with 5 litres during the measurement, and the remaining amount was for the specimen tray. As illustrated in Figure 1, the tray comprises 7x3 specimens, each of which may hold 50 mL of the allocated material.

The heater is turned on to a certain required temperature ranging between (-20 – 100) ºC once the specimen tray is fixed inside the tank and filled with the assigned material. The by-pass line helps to better mix the water throughout the heating process, resulting in temperature uniformity in the tank. Meanwhile, the data logger (Almemo 2890-9) with nine input channels, as shown in Figure 2, stores the information from the sensors. A data logger is a device that measures and displays signal voltages plotted with time or another signal voltage. Until the user stops the data recorder, it measures in a continuous stream mode, every time new samples are taken during the measurement, they are transferred straight to the computer, where they are processed, displayed, and, if desired, stored on disk. The presence of a change in an input signal is immediately apparent.

Two temperature sensors, NiCr-Ni type k (-40 →1000ºC), are located on the right and left sides of the tank, as well as one ambient temperature sensor used as a reference temperature to determine when the container cools down to near ambient temperature, and another sensor inside the PCM capsule, as well as two heat flux sensors in the internal and external parts of the insulation. Finally, the goal is to calculate the time required for each experiment in which the water cools down to near ambient temperature after being heated, which is represented by the equation:

$$T_{avg\_tank} - T_{amb} \leq +1\ ^{o}C$$

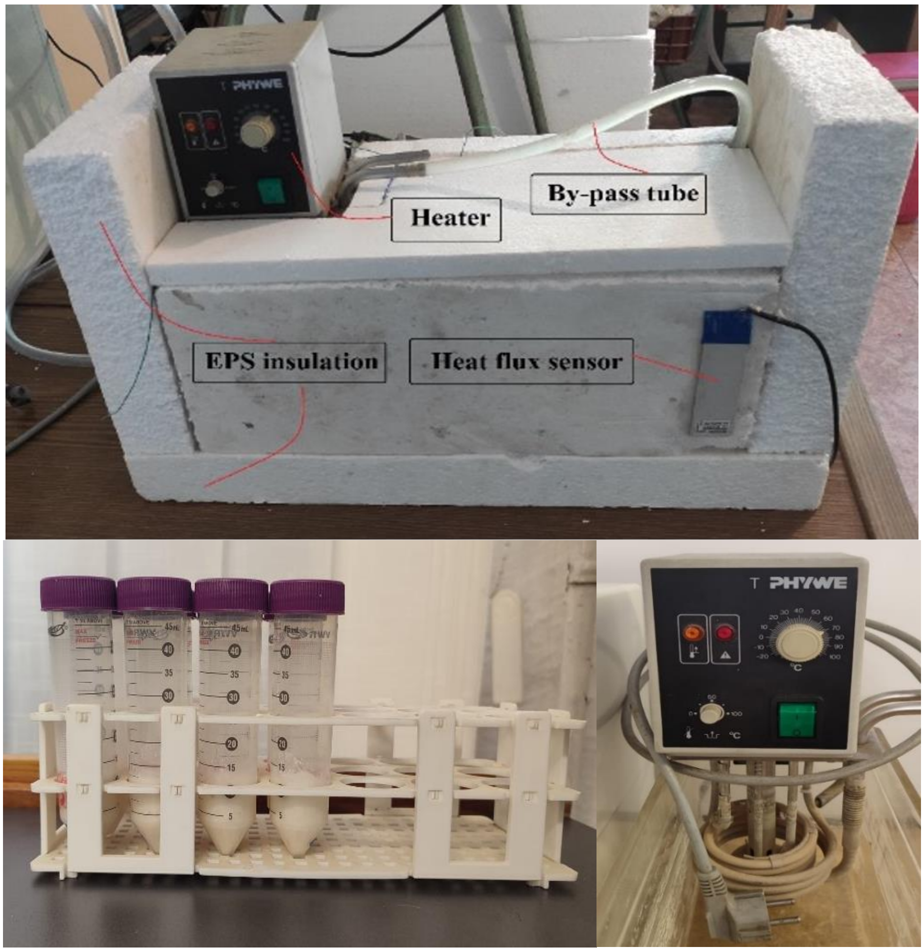

Figure 1. Experiment components

Figure 2. ALMEMO 2890-9 type data logger

The model was built using the R programming language, with coded values ranging from [-1, +1] for each variable, and the variables being "S" for Sample numbers, with the code (-1) for four samples and (+1) for eight samples. The quantity of PCM in each sample is denoted by the letter "Q," where (-1) code equals 5g and (+1) code equals 10g. Finally, the temperature is represented by the letter "T," with code (-1 denoting 45°C and code (+1) denoting 75°C. As seen in Figure , this results in a cube pattern, with each corner representing a set of these three variables, resulting in a single experiment. As indicated in Table 1, the number of experiments is determined using the form 2^k, where k is the number of variables; therefore, 23 results in eight measurements. To detect the second-degree non-linear coefficients, additional measurement was undertaken out of the cube borders at (S=6, Q=7.5g, T=60ºC) to add to the fact that the correlation was known before commencing the measurement.

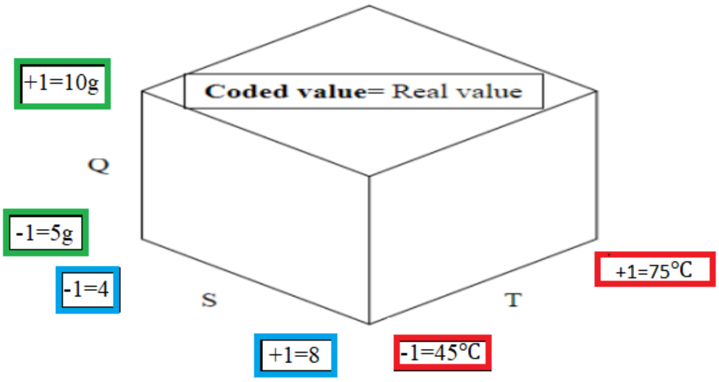

Figure 3. Experiment data set cube

The relationship between coded values and real values is shown by the following equation:

$$Coded\ value = \frac{Real\ value - Center\ point}{\frac{1}{2}(range)}$$

Table 1. Experiment parameters

	1	2	3	4	5	6	7	8	9_extra _pre
Samples [-]	-1	+1	-1	+1	-1	+1	-1	+1	0
Quantity [g]	-1	-1	+1	+1	-1	-1	+1	+1	0
Temperature [℃]	-1	-1	-1	-1	+1	+1	+1	+1	0

The method used is the least-squares method, which was developed by Carl Friedrich Gauss in 1795 and offered the overarching justification for the best-fit line placement among the data points. Iterations are used to solve the non-linear model in our experiment. The following coded equation represents the created model:

$$Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + u_i,\ i = 1..., n$$

Where:

- X_1, X_2 are the two independent variables (regressors)
- (Y_i, X_{1i}, X_{2i}) denote the ith observation on Y, X_1, and X_2.
- β_0 = unknown population intercept
- β_1 = effect on Y of a change in X_1, holding X_2 constant
- β_2 = effect on Y of a change in X_2, holding X_1 constant
- u_i = “error term” (omitted factors)

3. Results

In our experiment, the non-linear model is solved using iterations. The following coded equation represents the generated model:

$$y = 16.09 + 2.3S - 1.12Q + 0.2T - 1.3Q * S + 0.57S * T + 2.1Q * T + 0.2Q * S * T$$

To understand the relationship between the factors and the objective, a Pareto plot is conducted as in Figure 4.

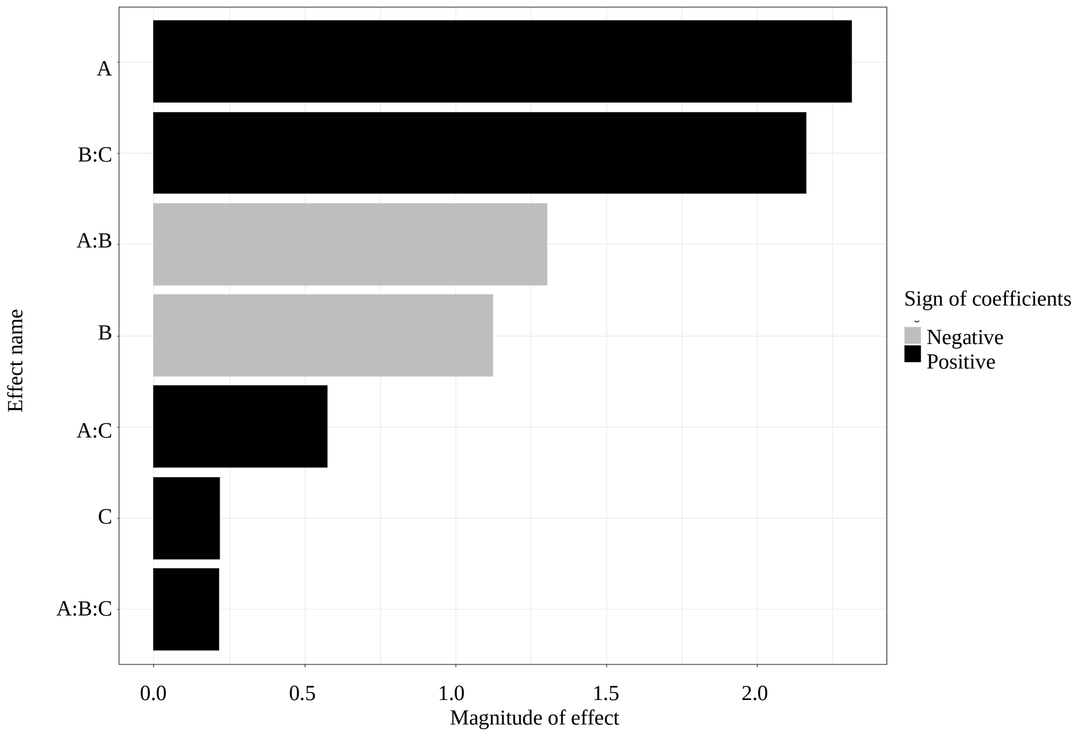

Figure 4. General model Pareto plot

A: Sample of number [-], B: Quantity [g], C: Temperature [℃]

The magnitude of each parameter is easily observed using the Pareto plot, while the sample numbers (A) and temperature with the quantity (BC) both have the biggest positive magnitude. In the third and fourth places, contrariwise, the sample with quantity both (AB) and the quantity coefficient (B) have a negative magnitude due to non-linear behaviour. In the end, the sample numbers with temperature (AC) have a low positive magnitude, adding to the fact that the 3-factor interaction S*Q*T (ABC) or any three-factor interactions do not exist in nature, but as it is shown in the Pareto plot Figure 4, it has almost zero magnitude. This is similar to temperature, which has a low influence. On the other hand, the contour plot of each two-factor interaction shows the curves where the overall result can be better, as seen in Figure 5.

Paying attention to some coded values that may have no meaning on the chart, for instance, S(A) = 0 or Q(B) = 0 matches 0 samples and 0 g, so below 3 values in the chart have no meaning in real-world values. As we can see in the chart, to increase the time, we should increase the temperature and the PCM quantity, as in the referring arrow. On the other hand, the coefficients of S, Q, and T in the equation show the plot's direction. In other words, a coded value of $\Delta T \rightarrow [+1]$ will add 0.2 hours to the overall result. Similarly, $\Delta Q \rightarrow [+1]$ will add -1.12 hours, and $\Delta S \rightarrow [+1]$ adds 2.3 hours, as in Figure 5.

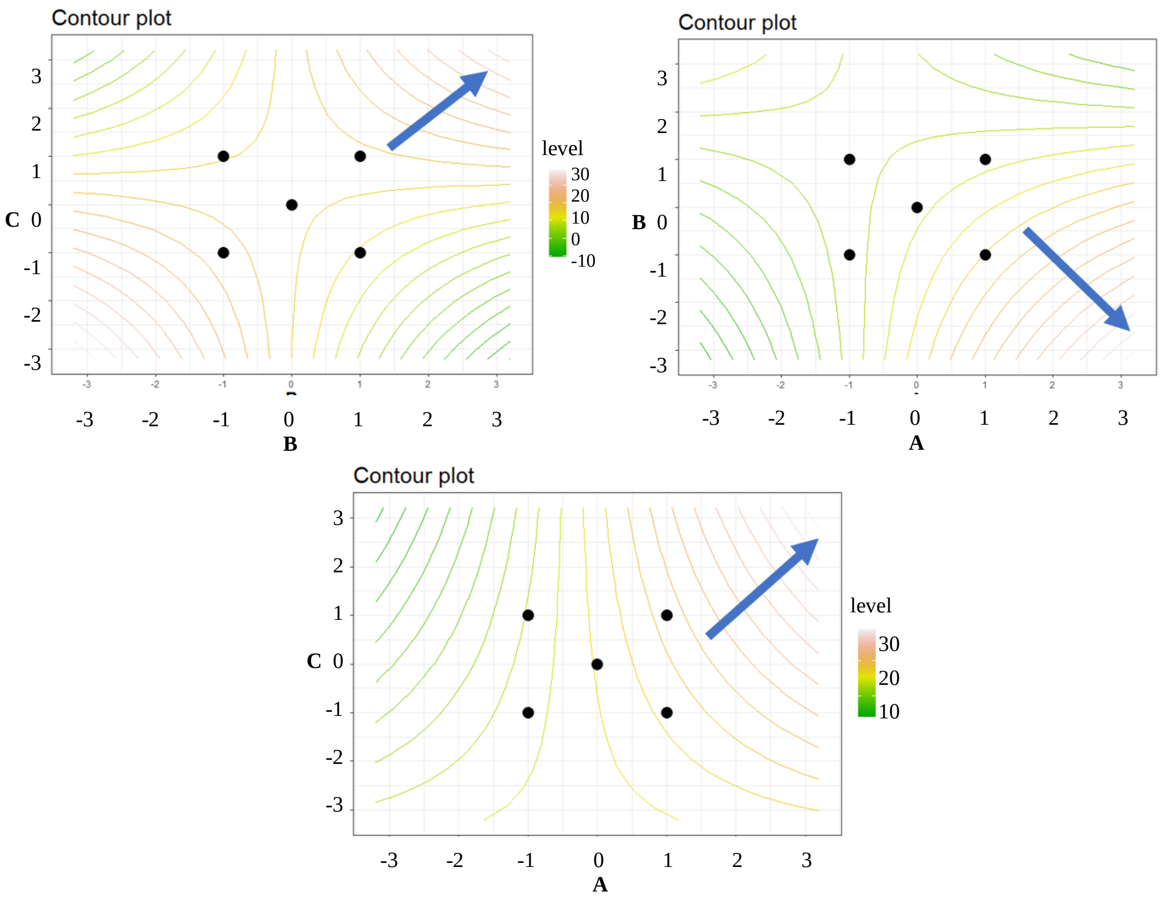

Figure 5. The two-factors interactions

On the other hand, the S*Q (AB) interaction shows better results (as shown in the5), we see for the efficiency B (quantity) is negative, and A (sample number) is positive. That means when the sample number increases, the time will be longer, and when the quantity decreases, the time will be longer. The black dots in the graph represent the 4 corners of the cube from a 2D Q*T perspective, adding to one pre and post experiments that were needed to identify the second-order coefficients. From the double-factor contour plot, the time increases if both factors are positive individually. That means both the sample number and the temperature should be increased to get the highest time.

By conducting eight trials to optimise the performance in a specific thermal tank, a matrix set of three variables was studied: temperature, quantity, and sample number of Soy wax. The mathematical first-degree, non-linear, 3-factors interaction equation was created using an R script based on the data. Furthermore, one extra experiment was to determine non-linear coefficients. Also, a Pareto Plot depicts the most influential elements in the non-linear equation, with sample numbers having the most significant magnitude, followed by temperature and PCM amount. The quantity has the strongest negative magnitude, indicating non-linearity. The results can be seen in Table 2.

Table 2. Experiment parameters

	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9_extra_pre**
Samples [S]	5	10	5	10	5	10	5	10	7.5
Quantity[Q]	4	4	8	8	4	4	8	8	6
Temperature[T]	45	45	45	45	75	75	75	75	60
y [hour]	16.3	11.43	12.5	12.56	11.43	19.38	15.68	19.28	15.03

4. Conclusions

Using domestic hot water can improve competitiveness and lead to environmental savings and economic advancement, especially in developing countries where solar energy is usually available and abundant. The predesigned solar hybrid systems have many advantages to be used nowadays, especially in developed countries where solar irradiation is usually sufficient. A big portion of energy is used in domestic hot water and industries and can be covered partially or totally with renewable energies such as solar energy. In practice, experimental work has been done in the lab where an insulated water tank filled with 5 litres of water and encapsulated PCM soy wax 68°C. Three parameters were applied: temperature, quantity, and sample number of Soy wax. This type of wax has not been tested before to see if it can be used in the solar system to prolong energy retention for a longer period. Especially when used in Hungary, where the heat loss is large. This article aims to study the impact of the size of the tank, its dimensions, the thickness of insulation, and its impact on solar fraction. An experimental work has been done in the laboratory using encapsulated PCM soy wax 68°C. Due to difficulties and time constraints, the three most important parameters were applied: temperature, quantity, and sample number. The response surface approach with non-linear correlation was used for the charging phase to determine the optimal number of samples and PCM quantity at two temperature levels. The results of temperature, sample numbers, and wax quantity demonstrate a 0.22, 2.3, and -1.12 [Hours] first-degree magnitude effect, respectively. In addition, each two-factor interaction contour plot shows the optimal directions of the optimised system. From the second-degree aspect, BC and AC have the highest impact on the system. This type of wax can play a significant role in saving energy in solar tanks in central European climates due to its low price, easy installation, and long-life service.

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